A Study on the Stability Analysis of a PWM Controlled Hydraulic Equipment

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PWM control, which is inherently nonlinear digital control, has been used for hydraulic equipment control because of the robustness and the availability of the low priced high speed on-off valve which is required for the system. Since this valve can be directly controlled without any D/A converter, it is easily implemented to hydraulic equipments with microcomputers. The objectives of this study is to analyze the limit cycle which ordinarily appears in the position control system using high speed on-off valve, and to give a criterion for the stability of this system. The nonlinear characteristics of PWM and cylinder friction are described by harmonic linearization and the effects of parameter variations to the system stability are investigated theoretically. Experimental results demonstrate the feasibility of the proposed method.

Key Words: Electro-Hydraulic Servo System, High Speed On-Off Solenoid Valve, PWM Method, Stability Analysis, Limit Cycle

Nomenclature

A_k : Piston area
c_d : Discharge coefficient
D : Duty (=acting time/T)
F_c : Coulomb's friction force
F_L : External load
F_R : Friction force of piston
G_k : Viscous damping coefficient
k_1 : Flow gain of valve
k_2 : Flow-pressure coefficient
k_f : Proportional gain
k_m : Maximum displacement of valve poppet
K : System gain ($=k_f \cdot k_m$)
K_q : Sizing factor of valve
L_s : Static friction
M : Piston mass
M_{RG} : Critical speed starting viscous damping friction
N_{PWM} : Describing function of PWM
N_s : Transfer function of valve-cylinder system
P_1, P_2 : Forward and return pressure of cylinder
P_S : Supply pressure
P_T : Tank pressure
q : Input signal of PWM
$\hat{q}$: Amplitude of PWM input signal
Q_1, Q_2 : Forward and return flows
q_{max} : Magnitude of PWM input signal of duty =1
T : Period of PWM carrier wave
T_q : Period of PWM input signal wave
T_S : Period of sampling time
$\hat{u}$: Amplitude of PWM output pulse
v : Velocity of piston
$\hat{v}$: Amplitude of piston velocity wave
V_t : Total volume of pipe, valve and cylinder chamber
w : Area gradient of valve port
x : Displacement of piston
x_{vi} : Displacement of valve poppet (i=a, b, c, d)
β_e : Effective bulk modulus of fluid and pipe
ϕ : Synchronizing degree
τ : Valve delay time

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$\tau(q)$: Width of PWM output
ω : Angular frequency of PWM input signal
$t_{on,dead}$: Switch on delay
$t_{on,disp}$: Switch on displacement time
$t_{off,dead}$: Switch off delay
$t_{off,disp}$: Switch off displacement time

1. Introduction

The control method using Pulse Width Modulation(PWM) is one of the nonlinear control schemes. Hydraulic equipments which are operated in PWM control usually use high speed on-off valves as control valves. The usage of this valve gives several advantages; this valve has very simple structure so that it has robustness to oil contamination and its price is low. And it can be easily implemented to the system by using microcomputer since the control of this valve can be carried out digitally without D/A converter (Wennmacher, 1992a, b; Wennmacher, 1994; Muto, et. al., 1988). Since the hydraulic cylinder system controlled by PWM has highly nonlinear characteristics such as the nonlinearity of PWM, the phase delay of valve poppet and the friction in the cylinder, the nonlinear characteristics should be considered in the analysis of the system stability. Tanaka(1988) considered this system as a linear time invariant discrete system and derived transfer function using z-transform, then, showed the stable limiting gain against the damping ratio on every sampling time. Noritsugu(1983) simplified the nonlinearity of PWM as a saturation function and considered it with the phase lag of valve poppet and the cylinder friction in the velocity control of pneumatic cylinder. But the nonlinearity of PWM signal generation is not considered fully. Prochnio(1986) utilized describing function based on the harmonic linearization in depicting the nonlinearity of PWM and considered the friction in cylinder, did not include the phase lag of the valve poppet.

The objective of this study is to present a method to predict the stability of hydraulic servo system. The electro-hydraulic position control system is theoretically modelled, which have high speed on-off valves controlled by PWM. In this process, the highly nonlinear characteristics of the system such as the dynamics of PWM, the phase lag of valve poppet and the cylinder friction are modelled mathematically. Especially the nonlinear characteristics of PWM and cylinder friction are expressed with describing function by the harmonic linearization. The validity of the presented method is investigated for the variation of the system parameters theoretically and experimentally.

2. Hydraulic Servo System Description and Analysis

2.1 Dynamic modelling

The schematic diagram of the hydraulic servo system employed in this study is shown in Fig. 1. This system consists of 4 high speed on-off valves and a hydraulic actuator(double rod double acting cylinder) as the main parts. The piston displacement(x), which is output of the system is feedbacked to the comparator, which is implemented by software inside the computer, to be compared with the reference input value. For the PWM operation with the high speed on-off valve, since the switching characteristic of valve poppet makes a significant influence to the performance of this system, the lag of the valve poppet movement should be considered in dynamic modelling. The hydraulic system is modelled in Fig. 2. When the ON signal is applied to the valves a and c, since the valves b and d receive the OFF signal simultaneously, the forward oil flow(Q_1) is oc-

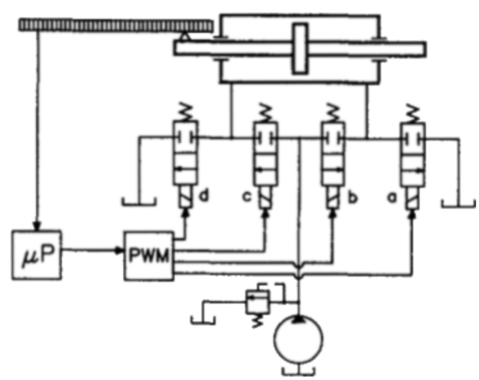

Fig. 1 Electro-hydraulic servo system with high speed on-off valves.

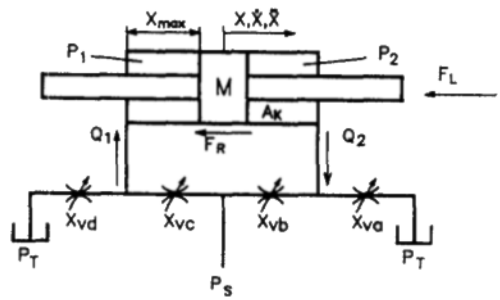

Fig. 2 Modelling diagram of hydraulic system.

curred through the valves a and c, and this makes the positive piston displacement. Under the assumption that the supply pressure(P_S) is constant, the leakage flows between a valve poppet and its seat and that of cylinder are negligible, and the valves are operated simultaneously, the flow rates(Q_1, Q_2) through the orifices of the high speed on-off valves are given by

$$Q_1 = c_d w \sqrt{\frac{2}{\rho}} \cdot x_{vc} \sqrt{P_S - P_1} \tag{1}$$

$$Q_2 = c_d w \sqrt{\frac{2}{\rho}} \cdot x_{va} \sqrt{P_2 - P_T} \tag{2}$$

where the load pressure(P_L) and the load flow (Q_L) are defined as

$$P_L = P_1 - P_2 \tag{3}$$

$$Q_L = \frac{Q_1 + Q_2}{2} \tag{4}$$

The load flow equation can be deriven as,

$$Q_L = K_q x_v \sqrt{P_S - P_L} \tag{5}$$

where

$$K_q = c_d w \sqrt{\frac{1}{\rho}}$$

The continuity equation of the valve-cylinder system is given by

$$Q_L = A_k \frac{dx}{dt} + \frac{V_t}{4\beta_e} \frac{dP_L}{dt} \tag{6}$$

The force balance equation of the piston is

$$M \frac{d^2 x}{dt^2} = A_k P_L - F_R \tag{7}$$

where the viscous damping friction of the piston is included in the term of total friction force of cylinder(F_R), and the external load(F_L) is omitted. The external force is neglected in the modelling, because it is generally not critical or of particular interest in system design, it cannot be altered appreciably by design, and it does not affect system stability. To derive the transfer function of the system, the load flow Eq. in (5) which is a nonlinear equation is linealized by the Taylor's expansion in the vicinity of an operating point(x_{v0}, P_{L0}) of interest, It can be written as (Merrit, 1967)

$$Q_L = k_1 x_v - k_2 P_L \tag{8}$$

2.2 Describing function of PWM

When a high speed on-off valve is operated in PWM mode, the input signal which is inflicted on the valve from the PWM module is a type of pulse row which has a constant pulse magnitude but a different pulse width and sign according to the magnitude of the input signal of PWM module. And the pulse row has the same interval between the pulse starting points. To investigate the stability of this system analytically, the characteristics of the PWM signal generation should be depicted analytically. Under the assumption that the on-off operations of the valves are symmetric and the linear part of this hydraulic system has the enough low pass filter function(Follinger, 1991), we derive the describing function of PWM signal generation which has the highly nonlinear characteristic. The PWM module catches the input signal(q) at every time interval and knows the end of the pulse width previously. The input signal ($q(t)$) of the PWM module can be represented as the following in the harmonic vibration balance state.

$$q(kT) = \hat{q} \cdot \sin(\omega \cdot kT + \phi) \tag{9}$$

where the synchronizing degree(ϕ) represents the phase lag between the first output pulse of PWM module and the input signal of PWM module in the harmonic vibration balance state. For example, the case of $\phi = 90°$ is shown in Fig. 3. The input signal $q(t)$ of PWM module which is depicted as Eq. (9) is a periodic signal which have a period(T_q). If the ratio between the period (T_q) and the period of PWM carrier wave(T) comes to $2n$ ($n = 1, 2, 3, \cdots$), Fourier series method can be utilized in calculation of the describing function of the PWM signal generation. The output pulse equation of PWM module is given by

$$u(t)=\begin{cases}\hat{u}\cdot\mathrm{sgn}(q(kT)) & kT\le t<kT+\tau[q(kT)]\\ 0 & kT+\tau[q(kT)]\le t<(k+1)T\end{cases} \tag{10}$$

where

$$\tau[q(kT)]=\begin{cases}T & D>1\\ T\cdot D & D\le 1\\ 0 & D<D_0\end{cases}$$

$u(t)$ is the function of the output signal of PWM module, $q(kT)$ is the Kth value of the input signal of PWM module, and D is a duty, which is determined as $|q(kT)/q_{\max}|$. D_0 is the duty value corresponds to the delay time in the valve operation. A vibration which continues steadily with a constant amplitude and frequency is called a limit cycle. Especially, when the ratio of (T_q/T) is 2 as shown in Fig. 3, we call it "1 pulse limit cycle." The Fourier coefficient a_1 and b_1 of basic vibration is given by

$$a_1=\frac{2\hat{u}}{\pi}\sin\left(\frac{\pi}{T}\cdot\tau(\hat{q})\right) \tag{11}$$

$$b_1=\frac{2\hat{u}}{\pi}\left[\cos\left(\frac{\pi}{T}\cdot\tau(\hat{q})\right)-1\right] \tag{12}$$

then the describing function of PWM signal generation is given as the following for the case of 1 pulse limit cycle.

$$N_{PWM}\left(\hat{q},1,\frac{\pi}{2}\right)=\frac{2\hat{u}}{\pi\hat{q}}\left[\sin\left(\frac{\pi}{T}\tau(\hat{q})\right)+j\left\{\cos\left(\frac{\pi}{T}\tau(\hat{q})\right)-1\right\}\right] \tag{13}$$

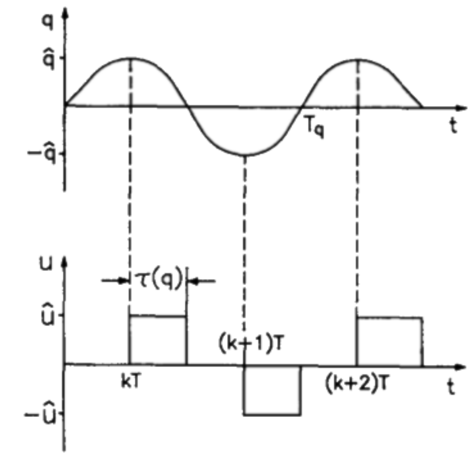

Fig. 3 Output pulse trains of 1 pulse limit cycle.

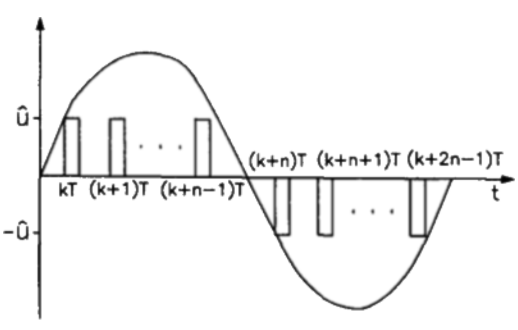

Fig. 4 Output pulse trains of n pulse limit cycle.

Meanwhile, the n pulse limit cycle has successively n positive pulses and n negative pulses for one period time(T_q) of limit cycle as shown in Fig. 4. The describing function of this case is derived as the following,

$$N_{PWM}(\hat{q},n,\phi)=\frac{2\hat{u}}{\pi\hat{q}}e^{-j\phi}\sum_{k=0}^{n-1}\{e^{-j\frac{\pi}{n}k}-e^{-j[\frac{\pi}{n}k+\frac{\pi}{nT}\tau(q(kT))]}\} \tag{14}$$

As shown in Eq. (14), $N_{PWM}(\hat{q},n,\phi)$ is a function of the amplitude$(\hat{q})$ of the input signal of PWM module, the pulse number(n) of limit cycle and synchronizing degree(ϕ), but it is independent to angular frequency(ω).

2.3 Describing function of friction characteristic

The friction characteristic of hydraulic cylinder which is modelled as shown in Fig. 5 consists of static friction, coulomb's friction and viscous damping friction which is dependent upon piston velocity. Supposing the hysterisis in friction characteristic is negligible, the friction characteristic shown in Fig. 5 can be formulated as the following: (Backé, 1992)

$$F_R(\hat{v})=G_k\cdot\hat{v}+\left[F_c+L_s\cdot\left(1-\frac{|\hat{v}|}{M_{RG}}\right)^4\right]\cdot\mathrm{sgn}(\hat{v}) \tag{15}$$

In order to derive the describing function, the above equation could be approximated as (Prochnio, 1986)

$$F_R(\hat{v}) = G_k \cdot \hat{v} + \left[F_c + \frac{L_s}{1+\left(\frac{\hat{v}}{c}\right)^2} \right] \cdot \mathrm{sgn}(\hat{v}) \tag{16}$$

where c is a constant required to approximate Eq. (15) to Eq. (16). In harmonic balance, the piston velocity $v(t)$ is given by

$$v(t) = \hat{v} \cdot \sin(\omega t) \tag{17}$$

the describing function of friction characteristic is

$$N_R(\hat{v}) = \frac{a_1(\hat{v})}{\hat{v}} \tag{18}$$

where $a_1(\hat{v})$ is the Fourier coefficient of basic vibration as follows:

$$a_1(\hat{v}) = \frac{1}{\pi}\int_0^{2\pi} \left\{ G_k \hat{v} \sin(x) + \left[F_c + \frac{L_s}{1+\left(\frac{\hat{v}\sin(x)}{c}\right)^2} \right] \mathrm{sgn}(\sin x) \right\} \cdot \sin(x)\, dx \tag{19}$$

Integrating Eq. (19) and substituting it into

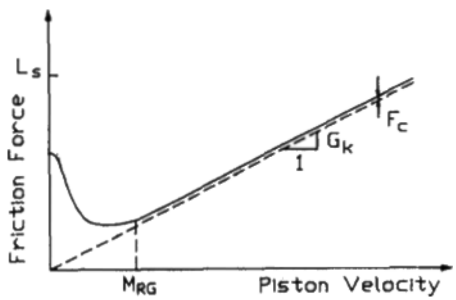

Fig. 5 Model of cylinder friction.

Eq. (18) the describing function of friction characteristic becomes as follows:

$$N_R(\hat{v}) = G_k + \frac{4F_c}{\pi \hat{v}} + \frac{2L_s \cdot c^2}{\pi \hat{v}^2 \sqrt{\hat{v}^2 + c^2}} \ln \left| \frac{\hat{v} + \sqrt{\hat{v}^2 + c^2}}{\hat{v} - \sqrt{\hat{v}^2 + c^2}} \right| \tag{20}$$

2.4 Analysis of the control loop

In order to investigate the stability of the system which have highly nonlinear characteristics such as PWM signal generation and friction in cylinder, the describing functions of these nonlinear characteristics are derived. Hence, the well known linear analysis method can be utilized. The above derived Eqs. (6), (7), (8), (14) and (20) are combined to construct the block diagram of the overall system as shown in Fig. 6. In this figure the nonlinear characteristics which is depicted by the describing functions are shown with double square frame. The transfer function N_S of the valve-cylinder system is derived as Eq. (21), the input of which is the valve spool displacement x_v and the output is cylinder displacement x,

$$N_S(\omega, \hat{v}) = \frac{c_1}{s(s^2 + c_2 s + c_3)} \tag{21}$$

where,

$$c_1 = \frac{k_1 4\beta_e A_k}{V_t M}$$

$$c_2 = \frac{4\beta_e k_2}{V_t} + \frac{F_R}{M}$$

$$c_3 = \frac{4\beta_e A_k^2}{V_t M}$$

The unstable phenomenon of a vibration system which has an element with a highly nonlinear characteristic usually starts with the limit cycle,

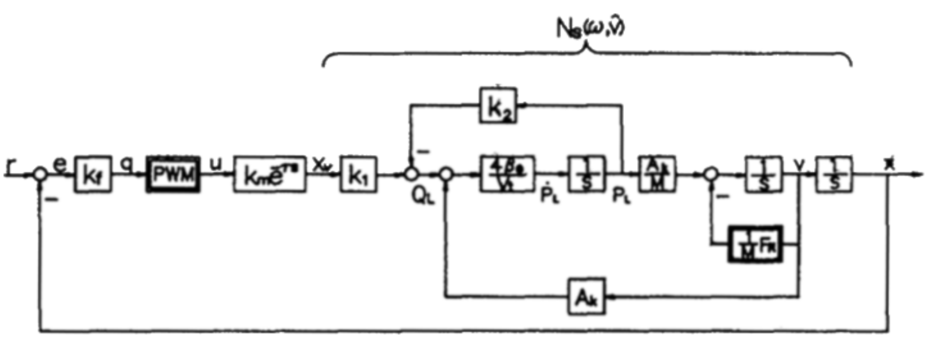

Fig. 6 Block diagram of PWM hydraulic cylinder system.

that is, the output of the system oscillate continuously with a constant amplitude and frequency. Whether the limit cycle will occur or not in a certain condition can be determined through the investigation of the solution of the closed loop characteristic equation. That is, in the harmonic vibration balance state, the limit cycle will occur if the solution of the closed loop characteristic equation exists (Prochnio, 1986; Follinger, 1991). The objective of this study is to present a method to predict the stability of the system so that it can be used to guarantee the stable operation in a wide range of operating conditions. The characteristic equation of the position control system shown in Fig. 6 is given as

$$1+Ke^{-\tau s}\cdot N_{PWM}\cdot N_S=0 \tag{22}$$

where K is obtained by the product of the proportional gain (k_f) and the maximum displacement of valve poppet (k_m). The characteristic Eq. (22) is so complicated that it is very difficult to solve it analytically. Therefore, a graphical method is utilized to get the solution of the characteristic equation.

3. Computer Simulation and Experiment

In order to solve the characteristic Eq. (22) by graphical method which is utilized in this study, the first step is to investigate the Eq. (22) and look for the unfixed parameters. In the second term of the left hand side of the Eq. (22) $N_{PWM}(\hat{q}, n, \phi)$ is the function of the amplitude ($\hat{q}$) of the input signal of PWM module, the pulse number (n) of limit cycle and synchronizing degree (ϕ), but it is independent to angular frequency (ω). $N_s(\omega, \hat{v})$ is the function of angular frequency (ω) and piston velocity amplitude ($\hat{v}$). The relation between the amplitude ($\hat{q}$) of the input signal of PWM module and piston velocity amplitude $\hat{v}$ is given by

$$\hat{q}=\frac{k_f}{\omega}\hat{v} \tag{23}$$

When this system is operated in harmonic vibration balance state, the ratio between the input signal period (T_q) and the period of PWM carrier wave (T) comes to $2n$ ($n=1, 2, 3, \cdots$), hence, the relation between angular frequency (ω) and PWM carrier wave period (T) is derived as the following:

$$\omega=\frac{\pi}{nT}, \qquad n=1, 2, 3, \cdots \tag{24}$$

In the above equation, when the pulse number (n) of limit cycle is fixed, angular frequency (ω) is uniquely determined. Therefore the unfixed parameters of Eq. (22) are the gain (K), the amplitude ($\hat{q}$) of the input signal of PWM module, synchronizing degree (ϕ) and pulse number (n) of limit cycle. At first to investigate whether the 1 pulse limit cycle occur or not, the pulse number of limit cycle is set as $n=1$. Then, we changed K value variously in the range of interest for the fixed ϕ and $\hat{v}$ value, otherwise we changed $\hat{v}$ value variously for the fixed ϕ and K value, and investigated the solutions which satisfy the Eq. (22). And for the cases of $n=2, 3, \cdots$ it is investigated in the same way. The values of each parameters which is used in computer simulation are shown in Table 1.

Table 1 System parameters used in computer simulation.

Parameters	Value	Dimension
A_k	7.65	cm^2
c	1.5	cm/s
F_c	0	kgf
L_s	21	kgf
G_k	5.46	kgf s/cm
k_1	4666.7	cm^2/s
k_2	0.3536	cm^5/kgf s
k_m	0.015	cm
M	5~80	kg
T	5	ms
T_s	1	ms
V_t	306	cm^3
V_d	50	cm^3
τ	1	ms
β_e	12000	kgf/cm^2

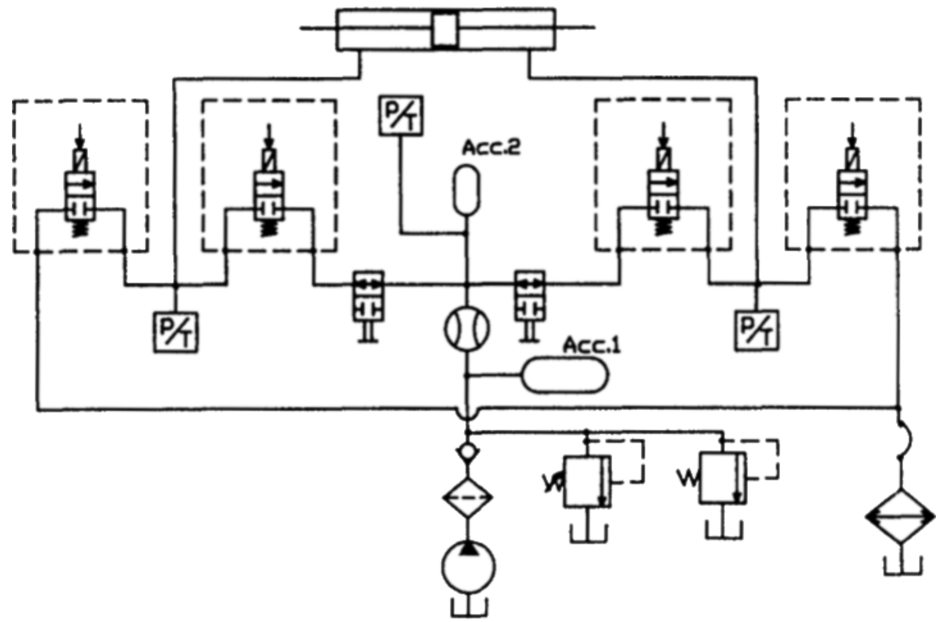

Fig. 7 Hydraulic circuit of experimental equipment.

The experimental apparatus is constructed as shown in Fig. 7, which has 4 high speed on-off valves, a double rod hydraulic cylinder, and a microcomputer as the major components. Piston displacement is measured by the digital optical sensor with $\pm 1\mu m$ accuracy. Pressure transducer is represented by P/T in Fig. 7, of which accuracy is ± 0.05 bar. It is an analog sensor so that the analog to digital converter is required to transfer measured values in the computer. The controller in control loop gets the sampled data on every millisecond, calculates the control law and gives it to 4 channel PWM modulator. PWM carrier frequency is 200 Hz and the period is 5ms. For the various inertia loads, we looked for the proportional gain that makes the system on the threshold from stability to instability with limit cycle. The major specifications of the components which is used in this experiment are shown in Table 2.

Table 2 Specification of experimental apparatus

Equipment		Specification
Hydraulic System	Electric Motor	5 PS
	Hydraulic Pump	$Q_{max}=8.9$ l/min
	Actuator	$A_k=7.65$ cm^2 $St.=10$ cm
	High speed On-off valve	$Q_{max}=4.2$ l/min $x_{vmax}=0.015$ cm $t_{on,dead}=0.46$ ms $t_{on,disp}=0.18$ ms $t_{off,dead}=0.30$ ms $t_{off,disp}=0.16$ ms
	Relief valve Filter Accmulator 1 Accumulator 2	$P_{max}=230$ kgf/cm^2 10 μm 4 liter 0.075 l, 70 bar
Electric Equipment	Displacement Transducer	12 bit(0.315mm)
Controller	Microcomputer	CPU386

4. Performance of the Control System

In the cases of inertia load 6kg and 50kg, the experimental results for step input are showed with the solid line and the dotted line respectively in Fig. 8 ($k_f=1.5$). The responses shows instability with almost regular oscillations which have almost constant amplitudes. For the inertia load 6kg, we can find that the response shows 1 pulse limit cycle because it vibrated with 100Hz frequency which have 10 peaks per 0.1 second, and

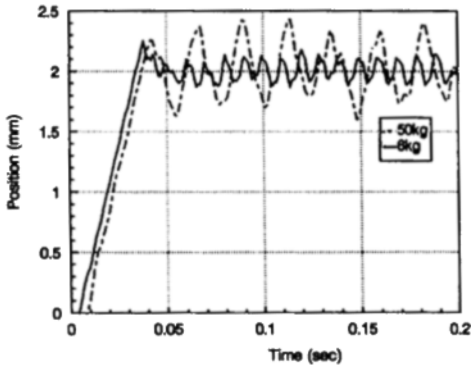

Fig. 8 Experimental results of n pulse limit cycle ($n=1, 2$).

this is the half of the PWM carrier frequency 200Hz. For the inertia load 50kg, the response shows almost 2 pulse limit cycle to have about 5 peaks per 0.1 second. Here the reason that the response does not show the exact 5 peaks per 0.1 second is guessed due to the following vibration of high speed on-off valve after its opening and closing operation. Figure 9 is one of the result figures which is utilized to investigate analytically the existence of 1 pulse limit cycle which is the solution of Eq. (22). One solid line of Fig. 9(a) is obtained for the case with the inertia load of 11kg, synchronizing degree(ϕ) of 88° and proportional gain(k_f) of 0.7, by plotting the value of $Ke^{-\tau s} \cdot N_{PWM} \cdot N_S$ in s-plane for the variation of the velocity amplitude($\hat{v}$) from 0.01 to 25, and the other solid line curves are obtained by repeating this process for the increased proportional gain(k_f) from 0.7 to 1.5 by step of 0.2.Similarly when the proportional gain(k_f) varies from 0.1 to 4.5 for the increased amplitude($\hat{v}$) from 0.2 to 3.2 by the step of 0.6, the values of $Ke^{-\tau s} \cdot N_{PWM} \cdot N_S$ are plotted with dotted lines. Herein, when the curve crosses the point (-1, 0) of s-plan, there exist the solution of Eq. (22) which is given by the values of the proportional gain(k_f) and velocity amplitude($\hat{v}$). One solid line of Fig. 9(b) is obtained for the case that inertia load is 11kg, synchronizing degree(ϕ) is 25° and proportional gain(k_f) is 2.0, by plotting the value of $Ke^{-\tau s} \cdot N_{PWM} \cdot N_S$ for the variation of the velocity amplitude($\hat{v}$) from 1 to 15. The other curves in Fig. 9(b) are obtained by repeating this process for

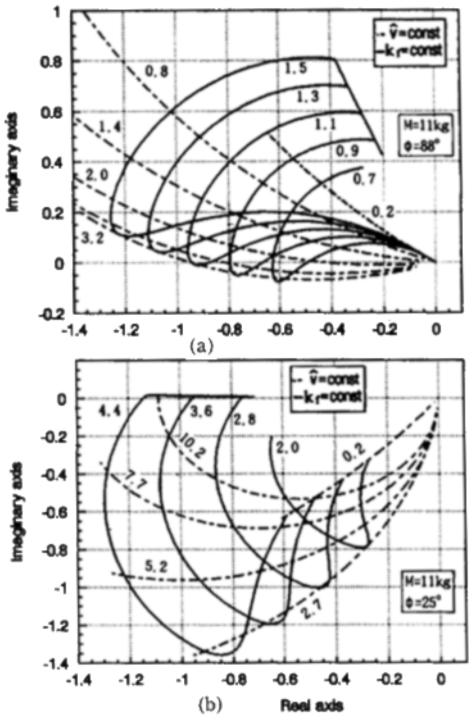

Fig. 9 Curves of $Ke^{-\tau s} \cdot N_{PWM} \cdot N_S$ for 1 pulse limit cycle.

the increased proportional(k_f) from 2.0 to 4.4 by the step of 0.8. Similarly when the proportional gain(k_f) varies from 0.1 to 4.5 for the increased velocity amplitude($\hat{v}$) from 0.2 to 10.2 by the step of 2.5, the values of $Ke^{-\tau s} \cdot N_{PWM} \cdot N_S$ are plotted with dotted lines. If the synchronizing degree(ϕ) is smaller than that of Fig. 9(b), since the whole curves turn counter clockwise, there is no curve crossing at the point(-1, 0) of s-plane and no solution of Eq. (22). Hence, in the condition of the inertia load 11kg the lower limit of synchronizing degree causing 1 pulse limit cycle is 25°. And if the synchronizing degree(ϕ) is bigger than that of Fig. 9(a), the whole curves turn clockwise and the upper limit of synchronizing degree can be determined. The analytical results which is acquired from the proposed method in this study are shown in Fig. 10.

In the region between the solid lines including ○ mark (inside of hatching lines), the solution of Eq. (22) exist and 1 pulse limit cycle is feasible. In the region between the dotted line includ-

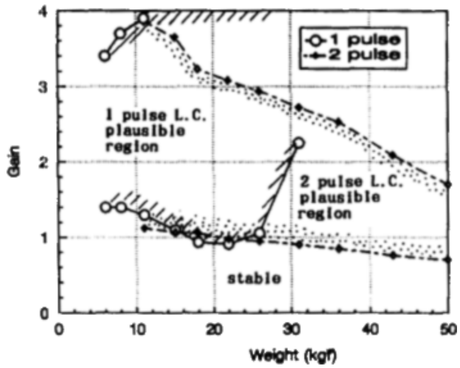

Fig. 10 The results of the stability analysis.

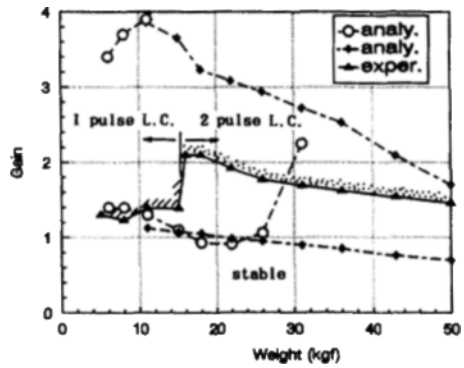

Fig. 11 The experimental result and analysis results.

ing ◇ mark (inside of the pointed dots), the solution of Eq. (22) exist and 2 pulse limit cycle is feasible, where ○ and ◇ mark shows the examination points. That is, we investigated the solution of Eq. (22) in the cases when piston masses are 6, 8, 11, 15, 18, 22kg··· etc. For the piston mass 6kg, 1 pulse limit cycle begin to occur from proportional gain $k_f=1.4$. But, if we vary the synchronizing degree(ϕ) from 90° to 36°, the upper limit of proportional gain(k_f) that cause 1 pulse limit cycle has changed to 3.4. In the upper region of the 1 pulse limit cycle feasibility area, the response is unstable with a mixed pulse limit cycle of 1 and 2 pulses. The mixed pulse limit cycle of 2 and 3 pulses occurs at the upper region of 2 pulse limit cycle feasibility area. We should design the controller so that the system to be operated in the stable area below the 1, 2 and n pulse limit cycle feasibility regions.

In Fig. 11, the solid line with the ▲ mark shows the experimentally obtained minimum proportional gain with which limit cycle occurs for the step input, and the analytical results of Fig. 10 are added for the comparison purpose. In this figure the stable limiting gains of the experimental result for inertia loads of 6kg and 8kg are as much lower as gain 0.1 times than those of analytical results, but the experimental results come into the analytical results fairly. In the region which is marked with hatching line in Fig. 11, up to the 15kg inertia load, the unstable phenomenon takes place with 1 pulse limit cycle. But in the region which is marked with pointed dot, from 16kg inertia load, 1 pulse limit cycle did not occur any more and the unstable phenomenon takes place with 2 pulse limit cycle. Further, we affirm the 3 pulse limit cycle to occur above 220kg mass through the method presented in this study. Therefore we can see 1 pulse limit cycle occur in small mass and n pulse limit cycle occurs according to the heaviness of the inertia load.

The stable limiting gain means the minimum proportional gain(k_f) in which limit cycle occurs for step input. The stable limiting gains, which are acquired through nonlinear simulation program(SIMULANT) (Backé, 1992) which is not used for system analysis but for dynamic simulation, are plotted with the solid line including ▼ mark in Fig. 12 with the results of stabitity analysis of Fig. 10 for comparison purposes. Herein ○ and ◇ marks are the examination points explained in Fig. 10 and ▼ marks are the examination points of simulation which are taken samely with those of stability analysis such as 5. 5, 8, 11, 15, 18, 22kg ··· etc. The simulaion results show a good correspondence with those of stability analysis. Comparing with experimental results of Fig. 11, the switching load point from 1 pulse limit cycle to 2 pulse moved about 7kg up. And the pulse change is appeared clearly because the gain value increased rapidly at the switching point. In simulation results, the unstable oscillation which appears with the mixed type of 1 pulse and 2 pulse limit cycle began at the 0.4 lower gain than the stable limiting gains of 2 pulse limit cycle shown in Fig. 12 from 31kg load. In Fig. 13 the results of linear anlysis which is derived in the basis of neglecting PWM characteristic and con-

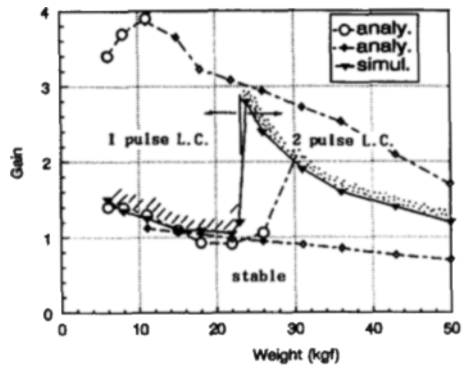

Fig. 12 The simulation results and analysis results.

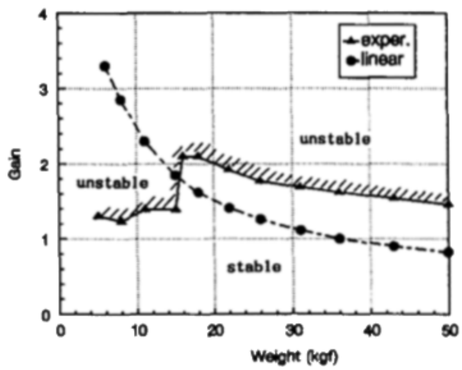

Fig. 13 The experimental result and the linear analysis result

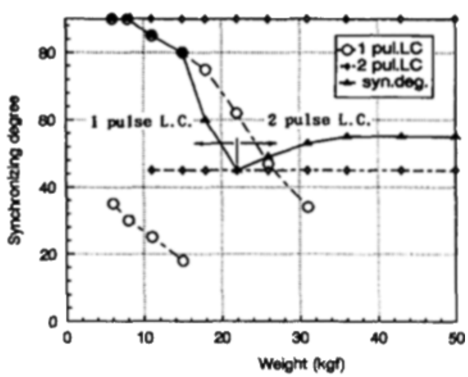

Fig. 14 The movement of the synchronizing degree.

sidering damping friction only in cylinder are shown with the experimental results (solid line including ▲ marks) of Fig. 11 for comparison purposes. The stable limiting gain of the linear analysis is demonstrated with the dotted line including ● marks. It is noticeable that the real value of stable limiting gains are far below in low inertia load than those of linear analysis which neglected PWM characteristic due to the occurrence of 1 pulse limit cycle.

According to the method suggested in this study, the range of synchronizing degree in which 1 pulse limit cycle is feasible is shown in Fig. 14 with the dotted line including ○ marks and 2 pulse limit cycle with ◇ marks. Based on the stable limiting gain of Fig. 12 obtained from the simulation, the feasible synchronizing degree are calculated by the method suggested in this study. In this figure, the trajectory of synchronizing degree moves along the line which has the minimum proportional gain up to 15kg inertia load. Then, it is separated and moved to the points to have increased limit gain. After the trajectory met with the lower limit line of 2 pulse limit cycle occurrence, the unstable phenomenon takes place with 2 pulse limit cycle and the stable limiting gain moves downward.

5. Conclusions

For the position control system of hydraulic equipments, high speed on-off valves controlled by PWM are used, in many practical cases. We investigated the stability of this system by the describing function method, considering the nonlinear characteristics such as PWM dynamics, friction in cylinder and the delay in the valve, and obtained the following conclusions:

(1) The stability of PWM position control system can be investigated analytically with the method suggested in this study using harmonic linearization for the highly nonlinear characteristics such as PWM and cylinder friction.

(2) The real stable limiting gain is smaller than that obtained from linear analysis neglecting PWM characteristic because of the occurrence of 1 pulse limit cycle.

(3) The unstable phenonenon of the PWM controlled system can be analyzed as n pulse limit cycle and its compound.

(4) 1 pulse limit cycle can occur with low inertia loads and according to the heaviness of the inertia load the pulse number of limit cycle increase.

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